

Chancellor Park Primary School
Written Methods and Calculation Policy 2014

At Chancellor Park Primary School we have drawn up a progressive programme identifying the range of strategies for recording and working out calculations which our pupils will be taught.

Children will move onto the next stage when they are competent at the method they are using. In the early stages repetition and consolidation is needed for children to understand place value and numbers. Therefore, all children will move at different rates and methods will be taught with that in mind rather than by the child's age / class.

This policy is intended to provide guidance to all staff on how to teach the four operations based upon providing pupils with a high-quality mathematics education and shares the aims of the national curriculum 'pupils becoming fluent in the fundamentals of mathematics, including varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately.' In addition to this the policy also recognises that pupils may need some methods to be simplified or extended to suit their learning needs.

This policy has been written using the Essex Education Service 'Calculation Guidance 2013/2014', STEP Cluster and Chancellor Park Primary School's previous calculation policy.

Addition Calculation Guidance

Year 1 - Pupils should be taught to:

- read, write and interpret mathematical statements involving addition ...equals (=) signs
- represent and use number ... within 20
- add ... one-digit and two-digit numbers to 20, including zero
- solve one-step problems that involve addition ..., using concrete objects and pictorial representations, and missing number problems.

Year 2 - Pupils should be taught to:

- Solve problems with addition...:
- using concrete objects and pictorial representations, including those involving numbers, quantities and measures
- applying their increasing knowledge of mental and written methods
- recall and use addition ... facts to 20 fluently, and derive and use related facts up to 100
- add ... numbers using concrete objects, pictorial representations, and mentally, including:
 - a two-digit number and ones
 - a two-digit number and tens
 - two two-digit numbers
 - adding three one-digit numbers
- show that addition of two numbers can be done in any order (commutative)
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems.

Year 3 - Pupils should be taught to:

- add ... numbers mentally, including:
 - a three-digit number and ones
 - a three-digit number and tens
 - a three-digit number and hundreds
- add ... numbers with up to three digits, using formal written methods of columnar addition
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition.

Year 4 - Pupils should be taught to:

- add ... numbers with up to 4 digits using the formal written methods of columnar addition ... where appropriate
- estimate and use inverse operations to check answers to a calculation
- solve addition... two-step problems in contexts, deciding which operations and methods to use and why.

Year 5 - Pupils should be taught to:

- add ... whole numbers with more than 4 digits, including using formal written methods (columnar addition)
- add ... numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition... multi-step problems in contexts, deciding which operations and methods to use and why.

Year 6 - Pupils should be taught to:

- perform mental calculations, including with mixed operations and large numbers.
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve addition... multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition
- use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.

STAGE 1

Pictures / multilink to join / count together



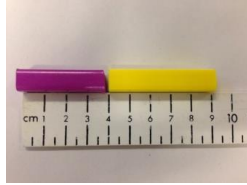
Count on using number tracks / number lines / 100 grids to support.



Develop concept of number bonds, initially to ten and then to 20.

Record related number facts.

e.g. $4 + 5 = 9$, $5 + 4 = 9$, $9 = 4 + 5$, $9 = 5 + 4$



Develop writing numbers alongside dots: $4 + 3$

.. .
.. .

STAGE 2

Develop understanding of the equals sign / equality and the concept of 'empty box' questions.

Record solutions to calculations such as $4 + \square = 6$.

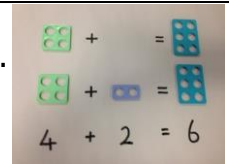
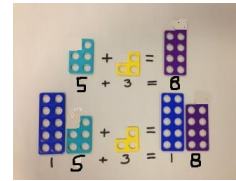
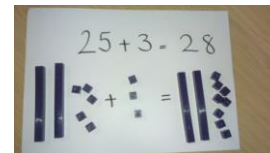
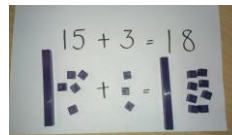
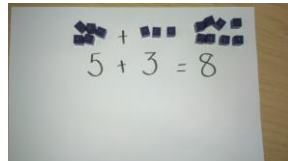
Use understanding of patterning, place value and partitioning to derive number facts.

e.g. $5 + 3 = 8$ (known fact)

TO + O

$$15 + 3 = 18$$

$$25 + 3 = 28$$



Begin to use understanding of place value and partitioning to carry out addition of one- digit and two-digit numbers.

STAGE 3 - *Practical apparatus is used to support this, as are number tracks /100 squares and number lines. Record the outcomes of calculations in horizontal format.*

TO + O

Continue to develop understanding of partitioning and place value and use this to support addition.

$$41 + 8$$

$$40 + 1 + 8$$

$$40 + 9 = 49$$



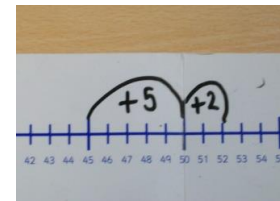
TO + O

Continue to develop addition by bridging to the next multiple of 10.

$$45 + 7$$

$$45 + 5 = 50$$

$$50 + 2 = 52$$



(STAGE 3 cont.)

TO + TO

Continue to develop understanding of partitioning and place value and use this to support addition.

$$25 + 32$$

$$20 + 30 = 50$$

$$5 + 2 = 7$$

$$50 + 7 = 57$$

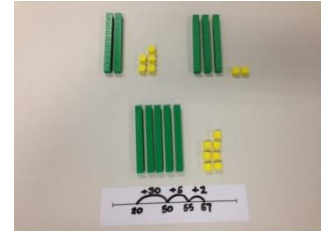
Where units combine to make totals greater than 10, regroup using partitioning skills

$$25 + 36$$

$$20 + 30 = 50$$

$$5 + 6 = 11$$

$$50 + 11 = 50 + 10 + 1 = 61$$

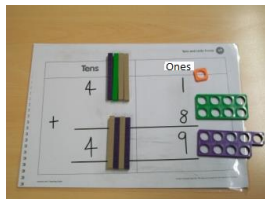


Pupils continue to determine when calculations are best carried out using mental strategies.

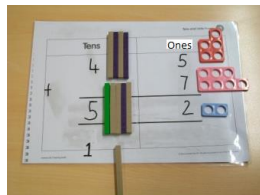
STAGE 4 - REMEMBER: When pupils confident with concepts of partitioning and place value, horizontal recording can be replaced with recording in columns with a focus on place value. *Resources may need to be used alongside this method initially.*

Teach carrying and only use expanded method if appropriate/necessary (it could be modelled by teacher to further explain why carrying is necessary. *Continue to determine when calculations are best carried out using mental strategies.*

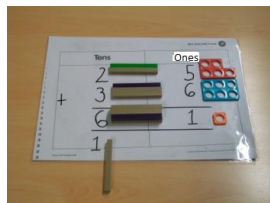
$$\begin{array}{r} \text{T O} \\ 4 \ 1 \\ + \ 8 \\ \hline 4 \ 9 \end{array}$$



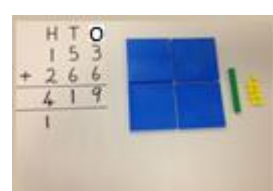
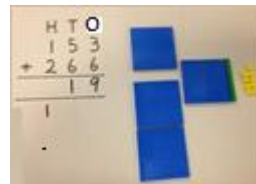
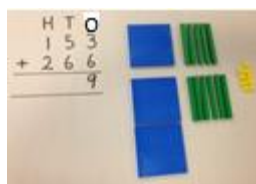
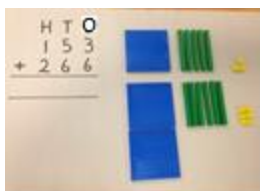
$$\begin{array}{r} \text{T O} \\ 4 \ 5 \\ + \ 7 \\ \hline 5 \ 2 \end{array}$$



$$\begin{array}{r} \text{T O} \\ 2 \ 5 \\ + \ 3 \ 6 \\ \hline 6 \ 1 \\ 1 \end{array}$$



$$\begin{array}{r} \text{H T O} \\ 1 \ 5 \ 3 \\ + \ 2 \ 6 \ 6 \\ \hline 4 \ 1 \ 9 \\ 1 \end{array}$$



Subtraction Calculation Guidance

Year 1

Pupils should be taught to:

- read, write and interpret mathematical statements involving ..., subtraction (-) and equals (=) signs
- represent and use number bonds and related subtraction facts within 20
- ... subtract one-digit and two-digit numbers to 20, including zero
- solve one-step problems that involve ... subtraction, using concrete objects and pictorial representations, and missing number problems such as $7 = \square - 9$.

Year 2

Pupils should be taught to:

- solve problems with... subtraction:
- using concrete objects and pictorial representations, including those involving numbers, quantities and measures
- applying their increasing knowledge of mental and written methods
- recall and use ... subtraction facts to 20 fluently, and derive and use related facts up to 100
- ... subtract numbers using concrete objects, pictorial representations, and mentally, including:
 - a two-digit number and ones
 - a two-digit number and tens
 - two two-digit numbers
 - adding three one-digit numbers
- show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot
- recognise and use the inverse relationship between addition and subtraction and use this to check calculations and missing number problems

Year 3

Pupils should be taught to:

- ... subtract numbers mentally, including:
 - a three-digit number and ones
 - a three-digit number and tens
 - a three-digit number and hundreds
- subtract numbers with up to three digits, using formal written methods of columnar subtraction
- estimate the answer to a calculation and use inverse operations to check answers
- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

Year 4

Pupils should be taught to:

- subtract numbers with up to 4 digits using the formal written methods of columnar, subtraction where apt
- estimate and use inverse operations to check answers to a calculation
- solve ..subtraction two-step problems in contexts, deciding which operations and methods to use and why.

Year 5

Pupils should be taught to:

- ... subtract whole numbers with more than 4 digits, including using formal written methods (columnar subtraction)
- ... subtract numbers mentally with increasingly large numbers
- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve ... subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

Year 6

Pupils should be taught to:

- perform mental calculations, including with mixed operations and large numbers.
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve ... subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.

STAGE 1

Using pictures take away the second number from the first number. 🍎🍎🍎🍎🍏🍏 (6-2)

Count back using number tracks / number lines / 100 grids to support the development of the concept of subtraction as take away.



Develop subtraction facts initially to ten and then to 20.
Record related number facts (and make links to related addition facts)
e.g. $9 - 4 = 5$, $9 - 5 = 4$



STAGE 2

Develop understanding of the equals sign / equality and the concept of 'empty box' questions.
Record solutions to calculations such as $9 - \square = 5$.

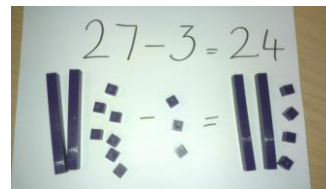
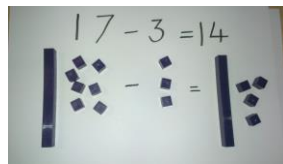
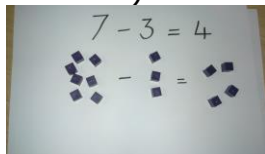
Use understanding of patterning, place value and partitioning to derive number facts.

e.g. $7 - 3 = 4$ (known fact)

TO - O

$$17 - 3 = 14$$

$$27 - 3 = 24$$



Begin to use understanding of place value and partitioning to support subtraction of one-digit and two-digit numbers.

STAGE 3 - Practical apparatus are used to support this, as are number tracks /100 squares and number lines. Record the outcomes of calculations in horizontal format. E.g. TO - O $27 - 6 = 21$

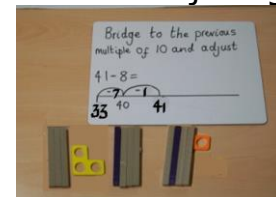
TO - O (Borrowing)

Continue to develop subtraction by bridging to the previous multiple of 10 and adjusting. This method builds on mental methods and knowledge of number-bonds.

$$41 - 8 = 33$$

$$41 - 1 = 40$$

$$40 - 7 = 33$$



TO - TO Also continue to develop understanding of partitioning and place value and use this to support subtraction.

$$95 - 34 = (60+1) = 61$$

$$5 - 4 = 1$$

$$90 - 30 = 60$$

*** (When borrowing of 2d - 2d numbers use STAGE 4) or use opposite for 'informal method'.

$$95 - 37 = 58$$

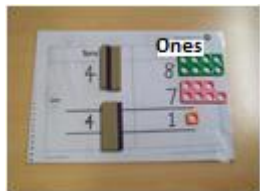
Pupils continue to determine when calculations are best carried out using mental strategies.

STAGE 4 – REMEMBER: When pupils confident with previous stages horizontal recording can begin to be replaced with recording in columns with a focus on place value. Resources may need to be used alongside these methods initially.

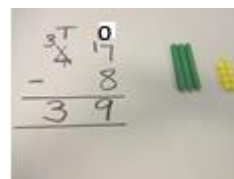
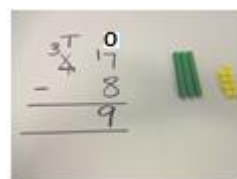
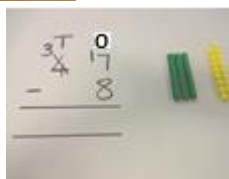
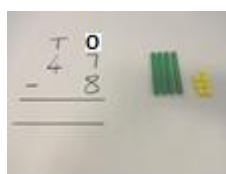
Only use expanded recording and apparatus to illustrate concept initially, if required, before moving towards the formal written method. (See **Appendix 1** – Subtraction)

Explore how the process relates to numbers with zeros as place holders

$$\begin{array}{r} \text{T} \quad \text{O} \\ 4 \quad 8 \\ - \quad 7 \\ \hline 4 \quad 1 \end{array}$$



$$\begin{array}{r} \text{T} \quad \text{O} \\ 4 \quad 7 \\ - \quad 8 \\ \hline 3 \quad 9 \end{array} \quad \longrightarrow \quad \begin{array}{r} \text{T} \quad \text{O} \\ \overset{3}{\cancel{4}} \quad \overset{1}{7} \\ - \quad 8 \\ \hline 3 \quad 9 \end{array}$$



TO – TO

$$\begin{array}{r} \text{T} \quad \text{O} \\ 3 \quad 6 \\ - \quad 2 \quad 5 \\ \hline 1 \quad 1 \end{array}$$

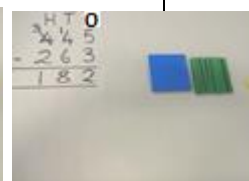
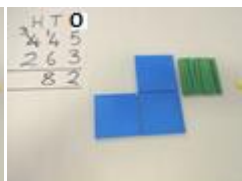
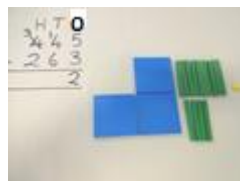
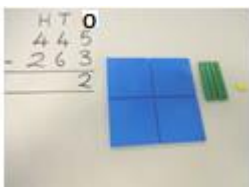
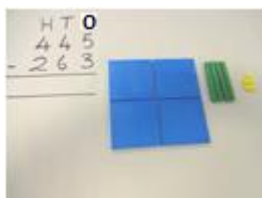
$$\begin{array}{r} \text{T} \quad \text{O} \\ 4 \quad 5 \\ - \quad 2 \quad 6 \\ \hline 1 \quad 9 \end{array} \quad \longrightarrow \quad \begin{array}{r} \text{T} \quad \text{O} \\ \overset{3}{\cancel{4}} \quad \overset{1}{5} \\ - \quad 2 \quad 6 \\ \hline 1 \quad 9 \end{array}$$

(See pictorial examples using TO – O subtraction above)

HTO - HTO

$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{O} \\ 3 \quad 3 \quad 6 \\ - \quad 2 \quad 2 \quad 5 \\ \hline 1 \quad 1 \quad 1 \end{array}$$

$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{O} \\ \overset{3}{\cancel{4}} \quad \overset{1}{4} \quad 5 \\ - \quad 2 \quad 6 \quad 3 \\ \hline 1 \quad 8 \quad 2 \end{array}$$



$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{O} \\ \overset{3}{\cancel{4}} \quad \overset{1}{\cancel{4}} \quad \overset{1}{5} \\ - \quad 2 \quad 6 \quad 9 \\ \hline 1 \quad 3 \quad 6 \end{array}$$

Multiplication Calculation Guidance

Year 1

Pupils should be taught to:

- solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Year 2

Pupils should be taught to:

- recall and use multiplication ... facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication ... within the multiplication tables and write them using the multiplication (\times), ... and equals (=) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- solve problems involving multiplication ... using materials, arrays, repeated addition, mental methods, and multiplication ... including problems in contexts.

Year 3

Pupils should be taught to:

- recall and use multiplication ... facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for multiplication ... using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written method
- solve problems, including missing number problems, involving multiplication ... including integer scaling problems and correspondence problems in which n objects are connected to m objects.

Year 4

Pupils should be taught to:

- recall multiplication ... facts for multiplication tables up to 12×12
- use place value, known and derived facts to multiply ... mentally, including: multiplying by 0 and 1; multiplying together three numbers
- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects.

Year 5

Pupils should be taught to:

- solve problems involving multiplication and division where larger numbers are used by decomposing them into their factors
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- multiply ... numbers mentally drawing upon known facts
- multiply ... whole numbers and those involving decimals by 10, 100 and 1000
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

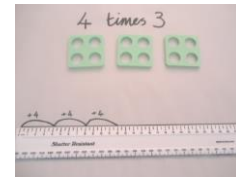
Year 6

Pupils should be taught to:

- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- perform mental calculations, including with mixed operations and large numbers.
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.

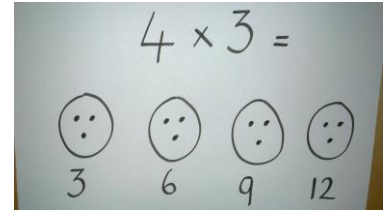
STAGE 1

Develop multiplication as repeated grouping (repeated addition of sets of the same size) using practical apparatus and diagrams.

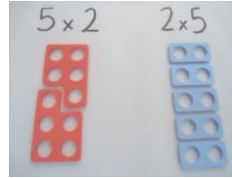
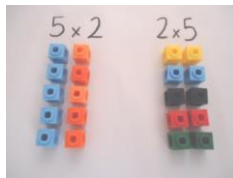


STAGE 2

Develop the use of x and = symbols to record calculations horizontally.

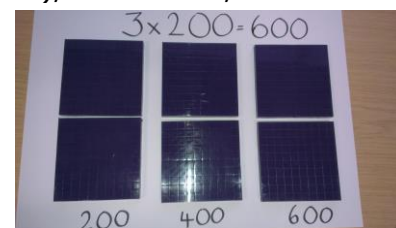
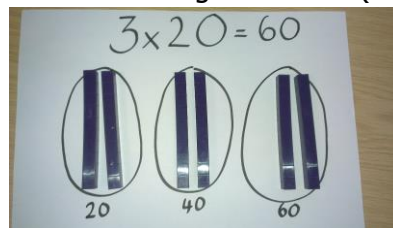
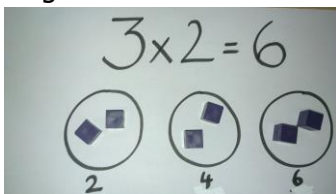


Use arrays and other practical apparatus to illustrate commutativity (that multiplication calculations can be carried out in any order) e.g. 2×5 arrives at the same product as 5×2 .



STAGE 3

Begin to derive new facts from known facts: e.g. $3 \times 2 = 6$ (known fact), $3 \times 20 = 60$, $300 \times 2 = 600$ etc.



STAGE 4

Begin to use understanding of place value and partitioning to carry out multiplication of two-digit by one-digit numbers

1). 15×4 $4 \times 5 = 20$
 $\swarrow \searrow$ $4 \times 10 = 40$
 10 5 $20 + 40 = 60$

Extend written approaches to HTO x O, then to Th,HTO x O. Illustrate using partitioning approaches as required

215×4 $4 \times 5 = 20$
 $\swarrow \searrow \searrow$ $4 \times 10 = 40$
 200 10 5 $4 \times 200 = 800$

$$20 + 40 + 800 = 860$$

*****Stage 4 - Extended written approaches to HTO x TO and Th, HTO x TO.**

$26 \times 13 = \underline{338}$ $3 \times 6 = 18, 3 \times 20 = 60, 6 \times 10 = 60, 10 \times 20 = 200$ ($18 + 60 + 60 + 200 = \underline{338}$)

$226 \times 13 = \underline{2938}$ $3 \times 6 = 18, 3 \times 20 = 60, 3 \times 200 = 600, 10 \times 6 = 60, 10 \times 20 = 200, 10 \times 200 = 2000$
($18 + 60 + 600 + 60 + 200 + 2000 = \underline{2938}$ ***** (See next page)**)

STAGE 5

Develop expanded recording in columns and then move to formal written method, using practical apparatus to support as required.

$$\begin{array}{r} \text{T} \quad \text{O} \\ 1 \quad 5 \\ \times \quad \underline{4} \\ \underline{6} \quad 0 \\ 2 \end{array}$$

$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{O} \\ 2 \quad 1 \quad 5 \\ \times \quad \underline{4} \\ \underline{8} \quad 6 \quad 0 \\ 2 \end{array}$$

*****PLEASE NOTE:** If a child cannot move forwards after using column multiplication for 2 digit x by 1 digit, or 3 digit by 1 digit, go back to Stage 4 and use the extended written approach before trying 2 digit x 2 digit column multiplication.

Develop expanded recording in columns and then move to formal written method of long multiplication, using practical apparatus to support as required.

$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{O} \\ \quad \quad 2 \quad 6 \\ \times \quad \underline{1 \quad 3} \\ \quad \quad 7 \quad 8 \\ \quad \quad \quad 1 \\ \underline{2 \quad 6 \quad 0} \\ \underline{3 \quad 3 \quad 8} \\ 1 \end{array}$$

$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{O} \\ 2 \quad 2 \quad 6 \\ \times \quad \underline{1 \quad 3} \\ \underline{6 \quad 7 \quad 8} \\ \quad \quad 1 \\ \underline{2 \quad 2 \quad 6 \quad 0} \\ \underline{2 \quad 9 \quad 3 \quad 8} \\ 1 \end{array}$$

Division Calculation Guidance

Year 1

Pupils should be taught to:

- solve one-step problems involving ... division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

Year 2

Pupils should be taught to:

- recall and use ... division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for ... division within the multiplication tables and write them using the division (\div) and equals (=) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- solve problems involving ... division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

Year 3

Pupils should be taught to:

- recall and use ... division facts for the 3, 4 and 8 multiplication tables
- write and calculate mathematical statements for ... division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- solve problems, including missing number problems, involving ... division, including integer scaling problems and correspondence problems in which n objects are connected to m objects.

Year 4

Pupils should be taught to:

- recall ...division facts for multiplication tables up to 12×12
- use place value, known and derived facts to ... divide mentally, including: dividing by 1;
- recognise and use factor pairs and commutativity in mental calculations

Year 5

Pupils should be taught to:

- solve problems involving ...division where larger numbers are used by decomposing them into their factors
- ...divide numbers mentally drawing upon known facts
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- ... divide whole numbers and those involving decimals by 10, 100 and 1000
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates.

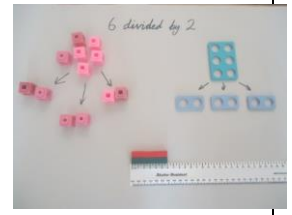
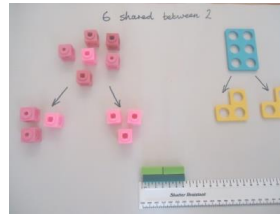
Year 6

Pupils should be taught to:

- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- perform mental calculations, including with mixed operations and large numbers.
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.

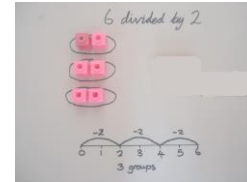
STAGE 1

Develop division as sharing.
(6 shared between 2 means "share 6 into 2 equal parts")
Notice the language / vocabulary.
(6 divided by 2 means "how many 2's are in 6?")



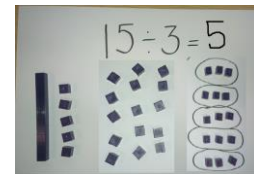
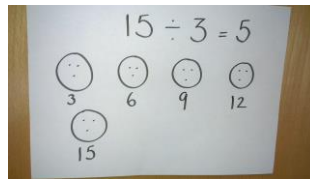
STAGE 2

Develop an understanding of division using arrays and number lines showing repeated groups.

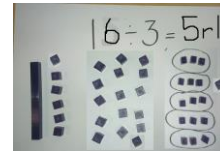
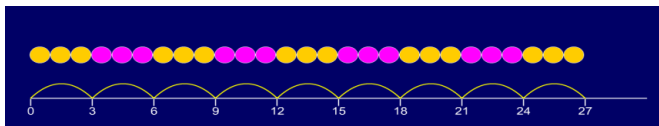


STAGE 3

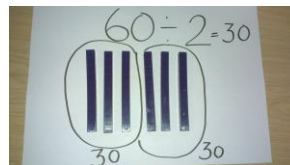
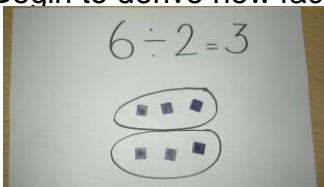
Develop the use of \div and $=$ symbols to record calculations horizontally
Use arrays and other practical apparatus, to illustrate making of repeated groups



Begin to carry out division of two-digit by one-digit numbers, first without remainders, then introducing remainders, illustrating this using informal methods first **if required**. $27 \div 3$

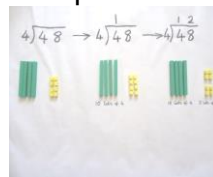
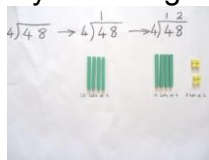


Begin to derive new facts from known facts, e.g. $6 \div 2 = 3$ (known fact), $60 \div 2 = 30$, $600 \div 2 = 300$

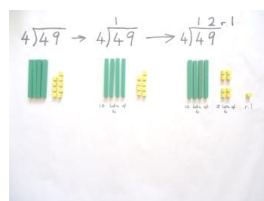
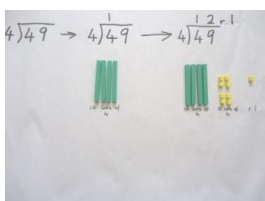


STAGE 4 - Move to develop the standard method for short division, first with no remainders, then with remainders, next carrying (no remainders), finally carrying (with remainders). **NOTE: written** $49 \div 4$
However formal layout changes to the 'bus stop' method.

$48 \div 4$



$49 \div 4$



$56 \div 4$

$57 \div 4$

STAGE 4 (cont..)

Extend written calculation methods to $HTO \div O$, then to $ThHTO \div O$, first with no remainders, then with remainders, illustrating this using informal methods first if required.

1). No carrying forward required

$$448 \div 4$$

(as above, but with additional hundreds column)

2). No carrying forward required, but with remainders

$$449 \div 4$$

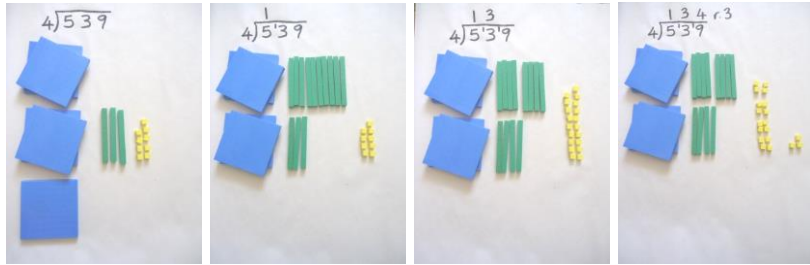
(as above, but with additional hundreds column)

3). Carrying forward required

$$536 \div 4$$

4). Carrying forward required, but with remainders

$$539 \div 4$$



Include: fractions as remainders and as decimals

e.g. $57 \div 4 = 14 \text{ r } 1$ (which is $14 \frac{1}{4}$)

$$\begin{array}{r} 14.25 \\ 4 \overline{) 57.00} \\ \underline{40} \\ 17 \\ \underline{16} \\ 10 \\ \underline{8} \\ 20 \\ \underline{20} \\ 0 \end{array}$$

STAGE 5

Extend written approaches to the formal method of long division when dividing by two-digit numbers.

$$\begin{array}{r} 28 \text{ r } 12 \\ 15 \overline{) 432} \\ \underline{30} \\ 132 \\ \underline{120} \\ 12 \end{array}$$

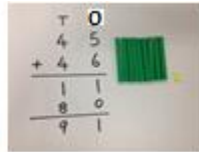
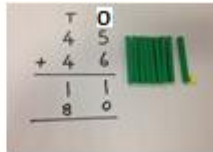
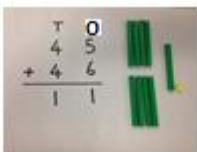
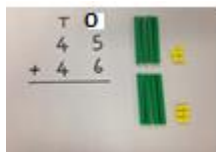
Include: fractions as remainders and as decimals

Appendix 1

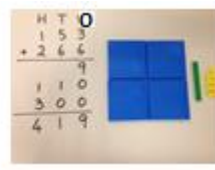
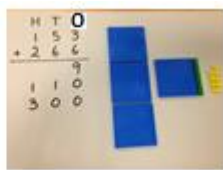
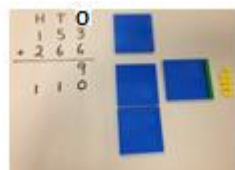
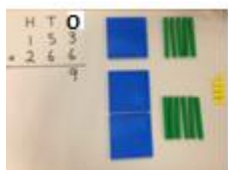
Addition

Only use expanded recording if appropriate to explore concept. The method could be modeled by the teacher to further explore the concept of addition.

$$\begin{array}{r} \text{T O} \\ 45 \\ + 46 \\ \hline 11 \\ 80 \\ \hline 91 \end{array}$$

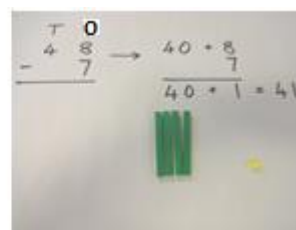
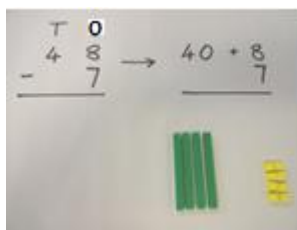


$$\begin{array}{r} \text{H T O} \\ 153 \\ + 266 \\ \hline 9 \\ 110 \\ 300 \\ \hline 419 \end{array}$$

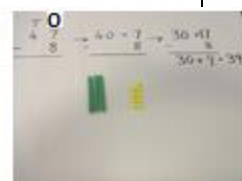
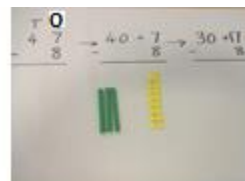
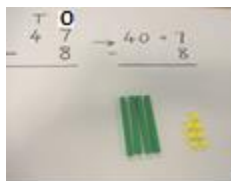


Subtraction

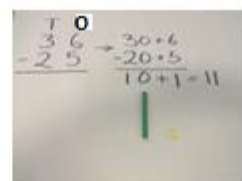
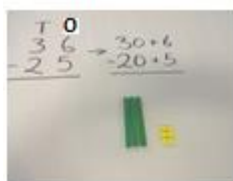
$$\begin{array}{r} \text{T O} \\ 48 \\ - 7 \\ \hline \end{array} \longrightarrow \begin{array}{r} 40 + 8 \\ - 7 \\ \hline 40 + 1 = 41 \end{array}$$



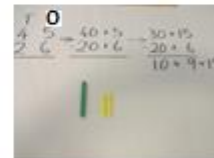
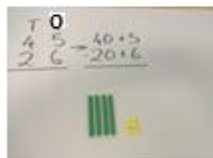
$$\begin{array}{r} \text{T O} \\ 47 \\ - 8 \\ \hline \end{array} \longrightarrow \begin{array}{r} 40 + 7 \\ - 8 \\ \hline \end{array} \longrightarrow \begin{array}{r} 30 + 17 \\ - 8 \\ \hline 30 + 9 = 39 \end{array}$$



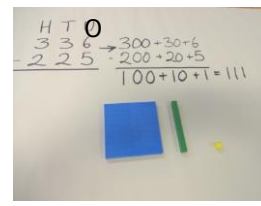
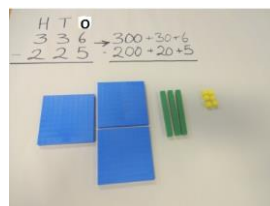
$$\begin{array}{r} \text{T O} \\ 36 \\ - 25 \\ \hline \end{array} \longrightarrow \begin{array}{r} 30 + 6 \\ - 20 + 5 \\ \hline 10 + 1 = 11 \end{array}$$



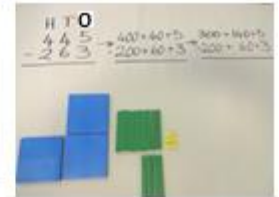
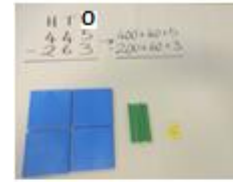
$$\begin{array}{r} \text{T O} \\ 45 \\ - 26 \\ \hline \end{array} \longrightarrow \begin{array}{r} 40 + 5 \\ - 20 + 6 \\ \hline \end{array} \longrightarrow \begin{array}{r} 30 + 15 \\ - 20 + 6 \\ \hline 10 + 9 = 19 \end{array}$$



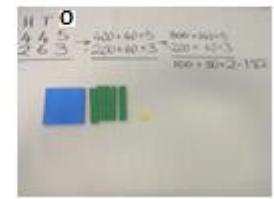
$$\begin{array}{r} \text{H T O} \\ 336 \\ - 225 \\ \hline \end{array} \longrightarrow \begin{array}{r} 300 + 30 + 6 \\ - 200 + 20 + 5 \\ \hline 100 + 10 + 1 = 111 \end{array}$$



$$\begin{array}{r}
 \text{H T O} \\
 445 \\
 - 263 \\
 \hline
 \end{array}
 \rightarrow
 \begin{array}{r}
 400 + 40 + 5 \\
 - 200 + 60 + 3 \\
 \hline
 \end{array}$$



$$\begin{array}{r}
 300 + 140 + 5 \\
 - 200 + 60 + 3 \\
 \hline
 80 + 2
 \end{array}
 \rightarrow
 \begin{array}{r}
 300 + 140 + 5 \\
 - 200 + 60 + 3 \\
 \hline
 100 + 80 + 2
 \end{array}$$



With place holders

$$\begin{array}{r}
 \text{H T O} \\
 405 \\
 - 269 \\
 \hline
 \end{array}
 \rightarrow
 \begin{array}{r}
 400 + 0 + 5 \\
 - 200 + 60 + 9 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 300 + 100 + 5 \\
 - 200 + 60 + 9 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 300 + 90 + 15 \\
 - 200 + 60 + 9 \\
 \hline
 100 + 30 + 3 = 133
 \end{array}$$

Multiplication

$$\begin{array}{r}
 \text{T O} \\
 15 \\
 \times \quad 4 \\
 \hline
 20 \quad (5 \times 4) \\
 40 \quad (10 \times 4) \\
 \hline
 60
 \end{array}$$

$$\begin{array}{r}
 \text{H T O} \\
 115 \\
 \times \quad 4 \\
 \hline
 20 \quad (5 \times 4) \\
 40 \quad (10 \times 4) \\
 800 \quad (200 \times 4) \\
 \hline
 860
 \end{array}$$

$$\begin{array}{r}
 \text{H T O} \\
 \quad 26 \\
 \times \quad 13 \\
 \hline
 \quad 18 \quad (6 \times 3) \\
 \quad 60 \quad (20 \times 3) \\
 \quad 60 \quad (6 \times 10) \\
 200 \quad (10 \times 20) \\
 \hline
 338 \\
 1
 \end{array}$$

	H	T	O	
	2	2	6	
x	1	3		
		1	8	(6 x 3)
		6	0	(20 x 3)
6	0	0	0	(200 x 3)
		6	0	(6 x 10)
2	0	0	0	(20 x 10)
2	0	0	0	(200 x 10)
2	9	3	8	
	1			

Division

	2	8	r 12
15)	4	3	2
	3	0	0
	1	3	2
	1	2	0
	1	2	